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Measuring the Agency Cost of Debt

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ABSTRACT

We adapt a contingent claims model of the firm to reflect the incentive effects of the capital structure and thereby to measure the agency costs of debt. An underlying model of the firm and the stochastic features of its product market are analyzed and an optimal operating policy is chosen. We identify the change in operating policy created by leverage and value this change. The model determines the value of the firm and its associated liabilities incorporating the agency consequences of debt.

THE OPTIMAL CAPITAL STRUCTURE for a firm is now widely regarded to be determined by a broad range of factors including a mix of tax effects, the various agency problems associated with different securities, and the various costs of issuing securities, including the costs created by adverse selection. While the existence of a theoretical optimum has been demonstrated in a variety of papers, a less explored area has been the construction of detailed models that enable us to measure each of the relevant factors for a particular company and thereby to determine the actual optimal mix for that firm. This gap in our understanding is particularly glaring in the case of agency costs. In order to allow a careful modelling of strategic relations, the parameters of most agency models are either so simplified that it is impossible to associate them with measurable parameters of a real world case, or else the models simply abstract from certain critical factors—such as a robust measure of price risk—that must be incorporated into any real application. For example, although we now understand that sinking funds, dividend restrictions, and other bond covenants help to resolve the conflict of interest between bondholders and equity, we do not yet have much in the way of models with which to determine the optimal parameters of these very covenants.

Contingent claims models can provide a consistent framework for multiperiod valuation that properly accounts for risk, but they usually abstract from the agency factors entering capital structure decisions. When using a contingent claims model to value a firm's securities it is common to take the value of the firm itself as governed by an exogenously defined stochastic process. The value of the firm's securities are then derived from this underlying value, and, as Merton (1974) points out in his paper on the pricing of risky debt, the Modigliani-Miller theorem obtains so that the value of the firm is independent of the value and the type of debt.

In order to apply the contingent claims techniques to a setting in which agency problems are central, some adaptation is necessary. In this paper we

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make the value of the firm an endogenous function of (1) an underlying stochastic variable describing the firm's product market and of (2) the management's choice of operating and investment decisions. The management maximizes the value of the equity claim as valued using a traditional contingent claims model, and this in turn determines the actual realized stochastic process that will describe the value of the firm and its debt. Different assumed financial structures will induce different operating strategies and therefore different realized stochastic processes for the value of the firm. The divergence of the chosen operating policy away from the first best operating policy gives rise to an agency cost of debt, and we are able to use the contingent claims model to precisely measure this cost and to identify how it varies with the underlying parameters of the model and with the relative profitability of the firm.

Earlier work on this type of problem includes Brennan and Schwartz (1984) and Fischer, Heinkel, and Zechner (1989). The former authors analyze the equity owners' optimal reinvestment and external financing policy over time given constraints imposed by pre-existing bond covenants. The latter consider the equity owners' optimal recapitalization policy in light of transaction costs and the tax benefits of debt, and they are able to then prescribe optimal ex ante call values to be included in the debt contract as it is originally written.

In this paper we focus on a specific production problem and analyze how the existence of debt directly changes the equity owners' choice of an operating policy for the business. The Brennan and Schwartz (1985) contingent claims model for valuing a mine is extended to incorporate the financial structure and to recognize the effects of the agency problems. An interesting benefit of using their model is that we are able to identify precisely the changes in the operating policy of the mine that are induced by the outstanding debt and thereby to directly relate the agency costs of debt with clearly suboptimal decisions in real production. The agency problems that arise are the underinvestment problem identified by Myers (1977), as well as the costs of bankruptcy. Our model measures these costs and thereby compares the tax benefits of debt with the agency costs of debt.

In Section I we extend the standard contingent claims model of the firm to incorporate the incentive effects of leverage on the firm's choice of operating policy. In Section II we apply this model to the task of measuring the agency costs. In Section III we present some numerical results and perform some analysis. We conclude in Section IV.

I. A Contingent Claims Valuation of a Mine in the Presence of Agency Costs

Brennan and Schwartz (1985) analyze a firm that owns a mine with a commodity inventory, Q. When the mine is open the commodity is extracted at a constant annual rate, q, and at a constant real average annual cost, a. When the mine is closed a constant real annual maintenance cost, m, is incurred. Corporate taxes are paid at rate τ on net income, and it is assumed that full offsets are allowed. At any point in time the mine can be closed at a real cost k_1 and reopened at a real cost k_2. The mine can also be costlessly abandoned.

Several crucial assumptions are made on the stochastic structure of the commodity price. First, the real spot price of the commodity, s, is determined in a competitive market and follows the exogenous process

$$ds = \mu s dt + \sigma s dz, \tag{1}$$

where dz is the increment to a standard Gauss-Wiener process; σ, the instantaneous standard deviation of the spot price, is assumed to be known and constant; and μ is the instantaneous drift in the real price. Second, it is assumed that there is a traded futures contract on the commodity. Then, following Ross (1978), if the convenience yield on the commodity is a constant proportion of the spot price, $\kappa(s) = \kappa s$, and if there exists a known constant real interest rate, r, the real price of a futures contract maturing in $\bar{t}$ periods is given by $f(s, \bar{t}) = se^{(r-\kappa)\bar{t}}$.

The market value of the mine, $v \equiv v(s, Q; j, \phi)$, is a function of the current commodity price, s, of the inventory, Q, of whether the mine is currently closed or open, $j = 1, 2$, and of the optimal operating policy, ϕ. An operating policy is described by three functions defining three critical commodity prices: $s_0(Q)$, the price, for a given inventory level, at which the mine is abandoned if it is already closed, $s_1(Q)$, the price for a given inventory level, at which the mine is closed if it was previously open, and $s_2(Q)$, the price, for a given inventory level, at which the mine is opened if it was previously closed, $\phi = (s_0, s_1, s_2)$. Throughout the remainder of the paper we suppress the argument Q, and write each of these functions simply as s_i, $i = 0, 1, 2$. The extraction rate for an open mine is assumed constant at q. Applying Ito's lemma of stochastic calculus the instantaneous change in the value of the mine is given by $dv = v_s ds + v_Q dQ + {}^1/_2 v_{ss}(ds)^2$. The cash flow from the mine is $[q(s-a)(j-1) - m(2-j)](1-\tau)$. Using an arbitrage argument similar to Black and Scholes (1973) the differential equation governing the value of the closed mine is written

$${}^1/_2\sigma^2 s^2 v_{ss}(s, Q; 1) + (r-\kappa) s v_s(s, Q; 1) - m(1-\tau) - rv(s, Q; 1) = 0, \tag{2}$$

and the value of the open mine is written

$$\begin{aligned} {}^1/_2\sigma^2 s^2 v_{ss}(s, Q; 2) + (r-\kappa) s v_s(s, Q; 2) - q v_Q(s, Q; 2) \\ + q(s-a)(1-\tau) - rv(s, Q; 2) = 0. \end{aligned} \tag{3}$$

Associated with this pair of equations are four boundary conditions:

$$v(s, 0; j) = 0, \tag{4}$$

$$v(s_0, Q; 1) = 0, \tag{5}$$

$$v(s_1, Q; 2) = \max\{v(s_1, Q; 1) - k_1(1-\tau), 0\}, \tag{6}$$

$$v(s_2, Q; 1) = v(s_2, Q; 2) - k_2(1-\tau). \tag{7}$$

The first best operating policy $\phi^{FB} \equiv (s_0^{FB}, s_1^{FB}, s_2^{FB})$ is characterized by the following first order conditions:

$$v_s(s_0^{FB}, Q; 1) = 0, \tag{8}$$

$$v_s(s_1^{FB}, Q; 2) = \begin{cases} v_s(s_1^{FB}, Q; 1) & \text{if } v(s_1^{FB}, Q; 1) - k_1(1-\tau) \geq 0 \\ 0 & \text{if } v(s_1^{FB}, Q; 1) - k_1(1-\tau) < 0, \end{cases} \tag{9}$$

$$v_s(s_2^{FB}, Q; 1) = v_s(s_2^{FB}, Q; 2). \tag{10}$$

solving equations (2) and (3) subject to boundary conditions (4)–(10), we derive simultaneously the first best value of the mine and the first best operating policy, v^{FB} and ϕ^{FB}.

If the firm is financed in part with debt, then the first best solution will not generally be chosen by managers acting in the interest of equity holders because the debt creates agency problems in the operation of the mine. The actual value of the firm will be determined by the operating policy chosen to maximize the value of the levered equity, the second best operating policy. To correctly value the firm and its associated liabilities we incorporate the effect of leverage into the Brennan and Schwartz (1985) model's derivation of the optimal operating policy.

We assume that the mine is financed in part with a bond described by a time path of the outstanding principal balance $P(t)$ and by a constant continuous coupon rate c. The interest payments on the bond, $cP(t)$, are tax deductible. We assume that there will be some point in time, T, such that $\forall t \geq T$, $P(t) = 0$, and we will call this the final maturity date of the bond. This covers a large range of possible debt structures. For example, a bond with constant amortization would satisfy the condition that $\forall t \leq T$, $P_t + cP = \delta$. A bond with a balloon payment at maturity can be approximated by a bond with zero principal payments until close to maturity, $\forall t < T - \varepsilon$, $P_t = 0$ and with continuous and quickly increasing principal payments as maturity approaches, $t > T - \varepsilon$ then $P_t \to \infty$ as $t \to T$. If T is much larger than the life of the mine, then this bond is comparable to a perpetuity.[1] When the assumed bond structure is very complicated it may have the appearance of a debt policy rather than of a single instrument. Our model values the firm given any assumed structure or policy. However, we do not allow the structure to be costlessly altered ex post to avoid bankruptcy. We solve for the optimal debt policy ex ante.

Before going back to revalue the levered mine it is necessary to value the equity since it is this value that will be maximized in choosing the firm's actual operating policy. The market value of the equity, $e \equiv e(s, Q, t; j, \phi')$, is a function of the current commodity price, s, of the inventory, Q, of the current time period, t, of whether the mine is currently closed or open, $j = 1, 2$, and of the modified operating policy, ϕ'. The modified operating

[1] For other payment structures that can be solved using the methodology of this paper see Mello and Parsons (1991).

policy acknowledges the right of the equity owners to default on the bond and is described by three functions defining three critical commodity prices: $s_d(Q,t)$ is the price, for any given inventory at time t, at which the equity owners default, while $s_1(Q,t)$ and $s_2(Q,t)$ are, as before, the prices, for any given inventory at time t, at which the mine is closed or opened, respectively, $\phi' = (s_d, s_1, s_2)$. Once again, we generally suppress the arguments Q and t, and write each of these functions simply as s_i, $i = d, 1, 2$. Upon default the firm is sold at its then current value, $v(s, Q, t; j)$ and these proceeds go to the bondholders. This possibility of default, of course, gives the equity owners a compound call option on the value of the mine. Consequently the value of the equity is time dependent and hence path dependent. Again, applying Ito's lemma, the instantaneous change in the value of the equity is given by $de = e_s ds + e_Q dQ + e_t dt + {}^1/_2 e_{ss}(ds)^2$. The cash flow from the equity is $[q(s-a)(j-1) - m(2-j)](1-\tau) + P_t - (1-\tau)cP(t)$. The last two terms are the principal payment and the after tax interest payment on the bond. Then the differential equation governing the value of the equity when the mine is closed is:

$$\begin{aligned} &{}^1/_2\sigma^2 s^2 e_{ss}(s,Q,t;1) + (r-\kappa)se_s(s,Q,t;1) + e_t(s,Q,t;1) \\ &\quad - m(1-\tau) + P_t - (1-\tau)cP(t) - re(s,Q,t;1) = 0, \end{aligned} \tag{11}$$

and when the mine is open is:

$$\begin{aligned} &{}^1/_2\sigma^2 s^2 e_{ss}(s,Q,t;s) + (r-\kappa)se_s(s,Q,t;2) - qe_Q(s,Q,t;2) \\ &\quad + e_t(s,Q,t;2) + q(s-a)(1-\tau) + P_t - (1-\tau)cP(t) \\ &\quad - re(s,Q,t;2) = 0. \end{aligned} \tag{12}$$

Associated with this pair of equations are also four boundary conditions:

$$e(s,0,t;j) = 0, \tag{13}$$

$$e(s_d,Q,t;1) = 0, \tag{14}$$

$$e(s_1,Q,t;2) = \max\{e(s_1,Q,t;1) - k_1(1-\tau), 0\}, \tag{15}$$

$$e(s_2,Q,t;1) = e(s_2,Q,t;2) - k_2(1-\tau). \tag{16}$$

The equity owner's optimal operating policy, $\phi'^P = (s_d^P, s_1^P, s_2^P)$ is characterized by the following first order conditions:

$$e_s(s_d^P,Q,t;1) = 0, \tag{17}$$

$$e_s(s_1^P,Q,t;2) = \begin{cases} e_s(s_1^P,Q,t;1) & \text{if } e(s_1^P,Q,t;1) - k_1(1-\tau) \geq 0 \\ 0 & \text{if } e(s_1^P,Q,t;1) - k_1(1-\tau) < 0, \end{cases} \tag{18}$$

$$e_s(s_2^P,Q,t;1) = e_s(s_2^P,Q,t;2), \tag{19}$$

The value of equity, e^P, and the optimal operating policy, $\phi'^P = (s_d^P, s_1^P, s_2^P)$, are derived simultaneously as the solution to the two differential equations (11) and (12) using boundary conditions (13)–(19).

To determine the value of the levered firm it is necessary to solve the pair of differential equations:

$$\tfrac{1}{2}\sigma^2 s^2 v_{ss}(s,Q,t;1) + (r-\kappa)sv_s(s,Q,t;1) + v_t(s,Q,t;1) - m(1-\tau) + \tau cP(t) - rv(s,Q,t;1) = 0, \quad (20)$$

and

$$\tfrac{1}{2}\sigma^2 s^2 v_{ss}(s,Q,t;2) + (r-\kappa)sv_s(s,Q,t;2) - qv_Q(s,Q,t;2) + v_t(s,Q,t;2) + q(s-a)(1-\tau) + \tau cP(t) - rv(s,Q,t;2) = 0. \quad (21)$$

along with boundary conditions based upon the operating policy that is optimal for the equity owners:

$$v(s,0,t;j) = 0, \quad (22)$$

$$v(s,Q,t;j) = v^{FB}(s,Q;j), \qquad \forall t \geq T \quad (23)$$

$$v(s_d^P,Q,t;1) = \alpha v^{FB}(s_d^P,Q;1), \quad (24)$$

$$v(s_1^P,Q,t;2) = \max\{v(s_1^P,Q,t;1) - k_1(1-\tau), 0\}, \quad (25)$$

$$v(s_2^P,Q,t;1) = v(s_2^P,Q,t;2) - k_2(1-\tau). \quad (26)$$

The value for the levered mine calculated using this system of equation is denoted v^P.

Boundary condition (24) requires some comment. Upon default the firm is put to the bondholder. The case in which the firm is subsequently operated according to the first best operating policy is equivalent to setting $\alpha = 1$. Another more general case incorporates the possibilities that either (1) there are costs of financial distress associated with bankruptcy, or (2) the bondholder cannot operate the firm and must reorganize it with a similar debt/equity structure—thereby reproducing the agency problem. This case is described by letting $\alpha \in [0,1)$. The parameter α measures the significance of the costs of financial distress, and as α approaches zero these agency costs increase.

The value for the outstanding bond is the difference between the total value of the mine and the value of the equity:

$$b^P = v^P - e^P. \quad (27)$$

To illustrate the model we calculated values for v^P, e^P, and b^P for a hypothetical example with input parameter listed in Table I. To our knowledge there is no closed-form solution to these various systems of equations. It is, however, possible to solve this system of equations using numerical methods as we have done for the hypothetical mine. The input parameters for our example are given in Table I. The critical commodity prices characterizing the equity owners' optimal operating policy at the initial inventory and at $t = 0$, $s_d^P(Q,t)$, $s_1^P(Q,t)$, and $s_2^P(Q,t)$, are displayed in Table II and contrasted with the critical commodity prices characterizing the first best operat-

Table I

Data for the Hypothetical Mining Firm

Total inventory in the ground:	Q = 150 million pounds
Annual real production for an open mine:	q = 10 million pounds
Average real production costs:	a = \$0.50/pound
Maintenance costs for a closed mine:	m = \$0.0/year
Closing and opening costs:	$k_1 = k_2$ = \$2 million
Real interest rate:	r = 2% annually
Commodity price variance:	σ^2 = 8% annually
Convenience yield:	κ = 1.5% annually
Corporate income tax rate:	τ = 34%

Table II

The Levered Firm's Choice of an Operating Policy

The Bond Contract	
par value	\$5.24 million
coupon rate	2%
annual debt service	\$0.4 million
final maturity date	15 years
Factor of firm's first best value at bankruptcy:	$\alpha = 0$

Critical Commodity Prices[a] (\$/pound)	First Best Operating Policy	Equity Owner's Optimal Operating Policy
abandonment/default	$s_0^{FB} = 0.00$	$s_d^P = 0.40$
closing	$s_1^{FB} = 0.59$	$s_1^P = 0.54$
opening	$s_2^{FB} = 0.84$	$s_2^P = 0.79$

[a] All values for the critical commodity prices in the optimal operating policies are calculated at the initial inventory given in Table I, 150 million pounds, and at $t = 0$.

ing policy at the initial inventory, $s_0^{FB}(Q)$, $s_1^{FB}(Q)$, and $s_2^{FB}(Q)$. The values for the levered firm, v^P, levered equity, e^P, and for the bond, b^P, are displayed in Table III.

II. Measuring the Agency Cost of Debt

It is important to note that in general the operating policy chosen to maximize the value of the equity claim will not be identical with the first best operating policy, $(s_d^P, s_1^P, s_2^P) \neq (s_0^{FB}, s_1^{FB}, s_2^{FB})$.[2] Without any agency costs of debt the value of the levered firm would be the first best value of the firm plus the interest tax shield of debt. Each added unit of debt increases the value of the firm by the value of its associated interest tax shields. The presence of agency costs modifies this. At first, with no debt outstanding, a

[2] More accurately, $\forall Q > 0$, $t < T$, $(s_d^P(Q,t), s_1^P(Q,t), s_2^P(Q,t)) \neq (s_0^{FB}(Q), s_1^{FB}(Q), s_2^{FB}(Q))$.

Table III

The Value of the Levered Firm and Its Liabilities

The Bond Contract	
par value	\$5.24 million
coupon rate	2%
annual debt service	\$0.4 million
final maturity date	15 years
Factor of firm's first best value at bankruptcy:	$\alpha = 0$

Commodity Price s	Firm Value $v^P(s,Q,t;j)$ (closed) $j=1$	Firm Value (open) $j=2$	Equity Value $e^P(s,Q,t;j)$ (closed) $j=1$	Equity Value (open) $j=2$	Bond Value $b^P(s,Q,t;j)$ (closed) $j=1$	Bond Value (open) $j=2$
0.05	0		0		0	
0.10	0		0		0	
0.15	0		0		0	
0.20	0		0		0	
0.25	0		0		0	
0.30	0		0		0	
0.35	0		0		0	
0.40	0		0		0	
0.45	2.71		0.32		2.39	
0.50	5.80		1.84		3.97	
0.55	8.80	6.90	4.14	3.45	4.66	3.46
0.60	12.08	10.79	7.01	6.83	5.07	3.96
0.65	15.44	15.26	10.30	10.69	5.14	4.57
0.70	19.15	19.58	13.93	14.81	5.22	4.73
0.75	23.03	24.07	17.97	19.08	5.16	4.99
0.80		28.52		23.44		5.09
0.85		32.99		27.84		5.15
0.90		37.46		32.28		5.18
0.95		41.92		36.72		5.20
1.00		46.40		41.18		5.22

marginal increase in debt has the same effect as before, increasing the value of the firm above the first best value by the size of the interest tax shields. As the size of debt increases, however, the marginal agency costs grow so that for large values of debt the total agency costs may far outweigh the total tax shields making the value of the levered firm less than the first best.

Our objective is to directly measure the agency costs associated with a particular financial structure. To do so we need to separate the effect of the tax shield for any outstanding bond: we define η as the value of the interest tax shield earned by the firm operated according to the policy ϕ'. The value of the interest tax shield is the solution to the following pair of differential equations:

$$\tfrac{1}{2}\sigma^2 s^2 \eta_{ss}(s,Q,t;1) + (r-\kappa)s\eta_s(s,Q,t;1) + \eta_t(s,Q,t;1) + \tau c P(t) - r\eta(s,Q,t;1) = 0, \quad (28)$$

and,

$$\tfrac{1}{2}\sigma^2 s^2 \eta_{ss}(s,Q,t;2) + (r-\kappa)s\eta_s(s,Q,t;2) - q\eta_Q(s,Q,t;2) + \eta_t(s,Q,t;2) + \tau c P(t) - r\eta(s,Q,t;2) = 0. \quad (29)$$

along with the boundary conditions:

$$\eta(s,0,t;j) = 0, \quad (30)$$

$$\eta(s,Q,t;j) = 0, \qquad \forall t \geq T \quad (31)$$

$$\eta(s_d^P,Q,t;1) = 0, \quad (32)$$

$$\eta(s_1^P,Q,t;2) = \eta(s_1^P,Q,t;1), \quad (33)$$

$$\eta(s_2^P,Q,t;1) = \eta(s_2^P,Q,t;2). \quad (34)$$

The agency costs of the debt $P(t)$ can then be defined as

$$\psi^P(s,Q,t;j) \equiv v^{FB}(s,Q;j) - [v^P(s,Q,t;j) - \eta^P(s,Q,t;j)]. \quad (35)$$

This is a precise measure of the value lost when the equity owners, because of the outstanding debt, change the operating policy from the first best.

Since the operating policy chosen to maximize the value of the equity is not the first best operating policy, the value of the levered firm is less than the first best value of the firm plus the interest tax shields, $v^p < v^{FB} + \eta^P$, the difference being the agency cost of debt. In Table IV the values for v^P are compared against the values for v^{FB} for the sample parameters described above, and the interest tax shields, η^P, and the agency costs of debt, ψ^P, are calculated.

The total agency costs reported in the tables are the consequence of three different changes in the firm's operating policy. First, the abandonment decision and its consequences is a straightforward example of the deadweight costs of financial distress that have been much discussed in the literature (see Shapiro and Titman (1986)). Second, the shareholders change the firm's policy for when to close the mine, $s_1^P < s_1^{FB}$, keeping it open longer than is Pareto optimal in the face of a falling commodity price. By spending the money to close the mine the firm saves on operating costs and preserves the limited inventory until the price rises again. However, the shareholders will bear the full expense of closure and do not enjoy the full benefits. They gamble that the commodity price may rise again without having fallen too far. In this case they will have avoided paying out of their own pockets the fixed cost of closure and then once again the fixed cost of reopening the mine. Third, the shareholders are also changing the firm's policy for when to reopen the mine, $s_2^P < s_2^{FB}$, opening it sooner than is Pareto optimal in the face of a rising commodity price. The shareholders have an interest in extracting the inventory as quickly as possible, since in the event of a future price decline they may have to put to the debtholders whatever remains of the inventory.

Table IV
The Agency Cost of Debt

The Bond Contract	
par value	$5.24 million
coupon rate	2%
annual debt service	$0.4 million
final maturity date	15 years
Factor of firm's first best value at bankruptcy:	$\alpha = 0$

Commodity Price s	First Best Firm Value $v^{FB}(s, Q, t; j)$ (closed) $j = 1$	First Best Firm Value (open) $j = 2$	Levered Firm Value $v^P(s, Q, t; j)$ (closed) $j = 1$	Levered Firm Value (open) $j = 2$	Tax Shields $\eta^P(s, Q, t; j)$ (closed) $j = 1$	Tax Shields (open) $j = 2$	Agency Costs: Absolute Value $\psi^P \equiv v^{FB} + \eta^P - v^P$ (closed) $j = 1$	Agency Costs: Absolute Value (open) $j = 2$	Agency Costs: % of First Best ψ^P / v^{FB} (closed) $j = 1$	Agency Costs: % of First Best (open) $j = 2$
0.05	0.00		0		0		0.00		100	
0.10	0.05		0		0		0.05		100	
0.15	0.25		0		0		0.25		100	
0.20	0.69		0		0		0.69		100	
0.25	1.39		0		0		1.39		100	
0.30	2.36		0		0		2.36		100	
0.35	3.60		0		0		3.60		100	
0.40	5.11		0		0		5.11		100	
0.45	6.90		2.71		0.136		4.33		62.7	
0.50	8.97		5.80		0.212		3.38		37.6	
0.55	11.31		8.80	6.90	0.244	0.244	2.75	3.33	24.3	33.3
0.60	13.94	12.67	12.08	10.79	0.257	0.257	2.12	2.13	15.2	16.8
0.65	16.86	16.14	15.44	15.26	0.262	0.261	1.68	1.14	10.0	7.1
0.70	20.07	20.04	19.15	19.58	0.264	0.264	1.18	0.73	5.9	3.6
0.75	23.59	24.18	23.03	24.07	0.266	0.265	0.82	0.38	3.5	1.6
0.80	27.40	28.47		28.52		0.266	0.46	0.22	1.7	0.8
0.85		32.84		32.99		0.266		0.12		0.4
0.90		37.26		37.46		0.266		0.07		0.2
0.95		41.70		41.92		0.266		0.05		0.1
1.00		46.15		46.40		0.266		0.02		0.0

III. Results

How significant are the agency costs of debt? Suppose that the current price of the commodity is \$0.80/pound. An open mine would have annual revenues at this price of \$8 million, annual costs of \$5 million, and a net cash flow after tax of \$1.98 million. If the firm operating an open mine has outstanding a bond with 15-year maturity and constant annual debt service payments of \$0.4 million we can see from Table III that its present value is \$28.52 million. The bond would have a market value of \$5.09 million, that is less than 18% of the firm value, a very low debt-to-value ratio. Moreover, the firm's current annual cash flow is five times its annual debt obligation. Clearly the probability of bankruptcy appears very small, and many would imagine that the agency costs of the debt should be correspondingly miniscule. From Table IV we read that the agency costs of this quantity of debt are \$0.22 million, or eight-tenths of a percent of firm value. In terms of the amount of debt sold, however, these agency costs are close to 4.3%, a very large value. This should be compared to other costs such as underwriting fees and administrative expenses which are usually 1.3% of the value of a debt offering according to Mikkelson and Partch (1986).

As mentioned earlier, this total agency cost is a combination of various factors—the suboptimal opening and closure policies and the dead weight cost of bankruptcy. To disentangle these causes and to explore the significance of the pure operational factors we reparameterize our model setting $\alpha = 1$ so that there are absolutely no dead weight costs associated with bankruptcy: the bondholders receive the first best value of the firm. The results for this case are displayed in Table V. When, as before, the current price of the commodity is \$0.80/pound the firm operating an open mine and with outstanding a bond with 15-year maturity and constant annual debt service payments of \$0.4 million has a present value of \$28.64 million. Not shown in the table, the bond would have a market value of \$5.20 million, again close to 18% of the firm value. The agency costs of debt in this case are \$0.10 million, about one half of the total agency costs from the previous example. The agency costs of debt amount to three-tenths of a percent of firm value or almost 2% of the value of debt sold.

Agency costs of this magnitude would certainly be an important determinant of the firm's capital structure decision even though the firm appears far from bankrupt. Table VI contains results also displayed in Figures 1 and 2. In Figure 1 we graph the levered firm's value as a function of its debt-to-value ratio. When the commodity price is \$0.65/pound and the mine is open the first best value of the firm is \$16.141 million. An outstanding 15-year bond with a 2% coupon and constant annual debt service payments totalling \$0.01 million offers tax shields worth \$0.007 million. The agency costs are \$0.005 million and the value of the levered firm is \$16.143 million. The bond is valued at \$0.228 million giving a debt-to-value ratio of less than 2%. Higher debt loads only lower the value of the firm since the marginal tax shields are less than the marginal agency costs of debt. When the commodity price is

Table V
The Agency Cost of Debt When Bankruptcy is Costless

The Bond Contract	
par value	\$5.24 million
coupon rate	2%
annual debt service	\$0.4 million
final maturity date	15 years

Factor of firm's first best value at bankruptcy: $\alpha = 1$

Commodity Price	First Best Firm Value $v^{FB}(s, Q, t; l)$		Levered Firm Value $v^{P}(s, Q, t; j)$		Tax Shields $\eta^{P}(s, Q, t; j)$		Agency Costs: Absolute Value $\psi^P \equiv \omega^{FB} + \eta^P - v^P$		Agency Costs: % of First Best ψ^P / v^{FB}	
s	(closed) $j = 1$	(open) $j = 2$	(closed) $j = 1$	(open) $j = 2$	(closed) $j = 1$	(open) $j = 2$	(closed) $j = 1$	(open) $j = 2$	(closed) $j = 1$	(open) $j = 2$
0.05	0.00		—[a]							
0.10	0.05		—							
0.15	0.25		—							
0.20	0.69		—							
0.25	1.39		—							
0.30	2.36		—							
0.35	3.60		—							
0.40	5.11		—							
0.45	6.90		6.58		0.136		0.45		6.6	
0.50	8.97		8.35		0.212		0.82		9.2	
0.55	11.31		10.47	8.60	0.244	0.244	1.08	1.63	9.5	16.3
0.60	13.94	12.67	13.13	12.11	0.257	0.257	1.07	0.81	7.7	6.4
0.65	16.86	16.14	16.19	15.86	0.262	0.261	0.93	0.55	5.5	3.4
0.70	20.07	20.04	19.60	20.03	0.264	0.264	0.73	0.27	3.6	1.4
0.75	23.59	24.18	23.28	24.26	0.266	0.265	0.75	0.19	2.4	0.8
0.80	27.40	28.47		28.64		0.266		0.10		0.3
0.85		32.84		33.06		0.266		0.05		0.2
0.90		37.26		37.50		0.266		0.03		0.1
0.95		41.70		41.95		0.266		0.02		0.0
1.00		46.15		46.41		0.266		0.01		0.0

[a] No values are shown for a levered firm when the commodity price is less than \$0.45/pound. If the price declines to \$0.40/pound, then the levered firm defaults. At this point the firm is reorganized and its value becomes the first best.

Table VI

The Optimal Quantity of Debt

The Bond Contract	
coupon rate	2%
final maturity date	15 years
Factor of firm's first best value at bankruptcy:	$\alpha = 0$

Panel A: Initial Commodity Price, s = \$0.65/pound

Fixed Annual Bond Payment Principal + Interest $[P(t) - P(t+1)] + cP(t)$	First Best Firm Value $v^{FB}(s, Q, t; 2)$	Tax Shields $\eta^P(s, Q, t; 2)$	Agency Costs $\psi^P(s, Q, t; 2)$	Levered Firm Value $v^P(s, Q, t; 2)$
0.000	16.141	0.000	0	16.141
0.005		0.003	0.001	16.143
0.010		0.007	0.005	16.143
0.015		0.010	0.013	16.138
0.020		0.013	0.024	16.130
0.025		0.017	0.030	16.128
0.030		0.020	0.035	16.126
0.035		0.023	0.041	16.123
0.040		0.027	0.074	16.094
0.045		0.030	0.081	16.090
0.050		0.033	0.107	16.067
0.1		0.067	0.395	15.813
0.2		0.133	0.787	15.487
0.3		0.197	1.142	15.196
0.4		0.261	1.144	15.258
0.5		0.322	1.470	14.993
0.6		0.378	1.466	15.053
0.7		0.432	2.012	14.561
0.8		0.486	2.046	14.581
0.9		0.487	2.494	14.134
1.0		0.503	3.038	13.606

Panel B: Initial Commodity Price, s = \$1.00/pound

Fixed Annual Bond Payment Principal + Interest $[P(t) - P(t+1)] + cP(t)$	First Best Firm Value $v^{FB}(s, Q, t; 2)$	Tax Shields $\eta^P(s, Q, t; j)$	Agency Costs $\psi^P(s, Q, t; 2)$	Levered Firm Value $v^P(s, Q, t; 2)$
0.0	46.152	0.000	0	46.152
0.5		0.333	0.036	46.449
1.0		0.660	0.146	46.691
1.5		0.974	0.442	46.685
2.0		1.243	1.686	45.709
2.5		1.462	2.698	44.916
3.0		1.610	6.697	29.690
3.5		1.615	10.497	30.237
4.0		1.381	20.849	23.224
4.5		1.063	27.867	18.249

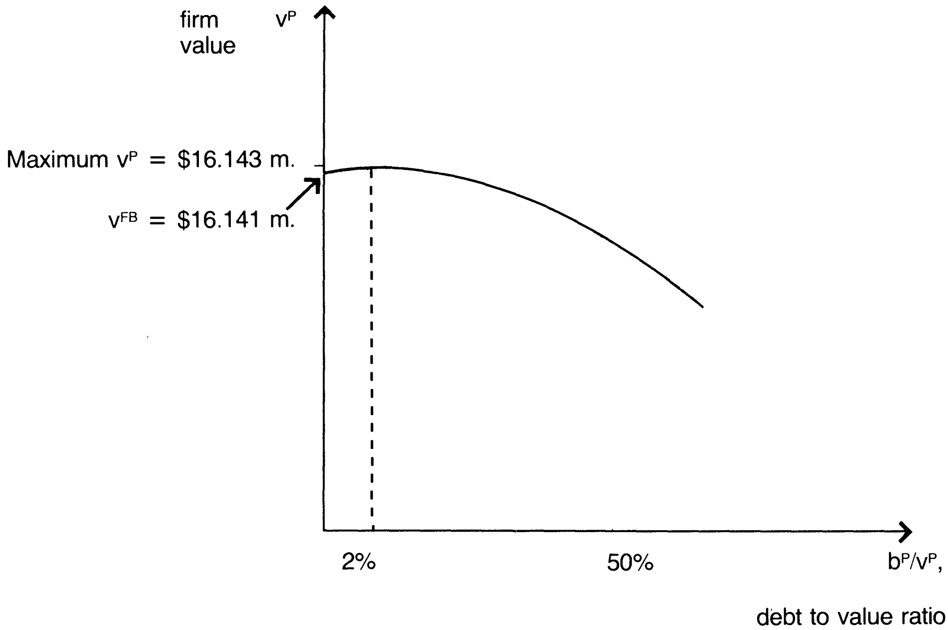

Figure 1. The effect of debt on the value of the firm. The value of a levered firm owning a mine with parameters specified in Table I. The current commodity price is $0.65/pound. The bond outstanding has a maturity of 15 years and a coupon rate of 2%. As the constant annual debt service payments are increased the debt-to-value ratio increases. The value of the levered firm initially increases due to the marginal benefits of interest tax shields. As the debt-to-value ratio further increases the marginal agency costs rise and the value of the firm begins to fall.

higher, $1.00/pound, the marginal agency costs are lower. This can be seen in Figure 2 where the optimal debt-to-value ratio is much higher. The first best value of the firm is $46.152 million. The interest tax shields associated with a 15-year 2% bond with annual debt service payments of $1.0 million raises the firm value to $46.691 million. The tax shields are valued at $0.66 million and agency costs equal to $0.146 million. The debt to value ratio is now just under 28%. Higher debt ratios lower the value of the firm overall.

It is interesting to explore the consequences of varying the structure of the debt on the total agency costs and the value of the firm. A popular rule of thumb is to match the maturity structure of the firm's liabilities to the maturity structure of its assets. In the case at hand, this rule of thumb would suggest that a constantly amortized bond matching the constant extraction rate of the mine and with a maturity matching the life of the inventory of the mine should be optimal. However, this is not necessarily correct. The maturity structure of the assets is very complicated, and ultimately depends upon the stochastic nature of the commodity price and the operating options available to the firm. These in turn depend upon the capital structure of the firm and the operating policy it induces. To highlight the simultaneity we contrast the results of two numerical examples.

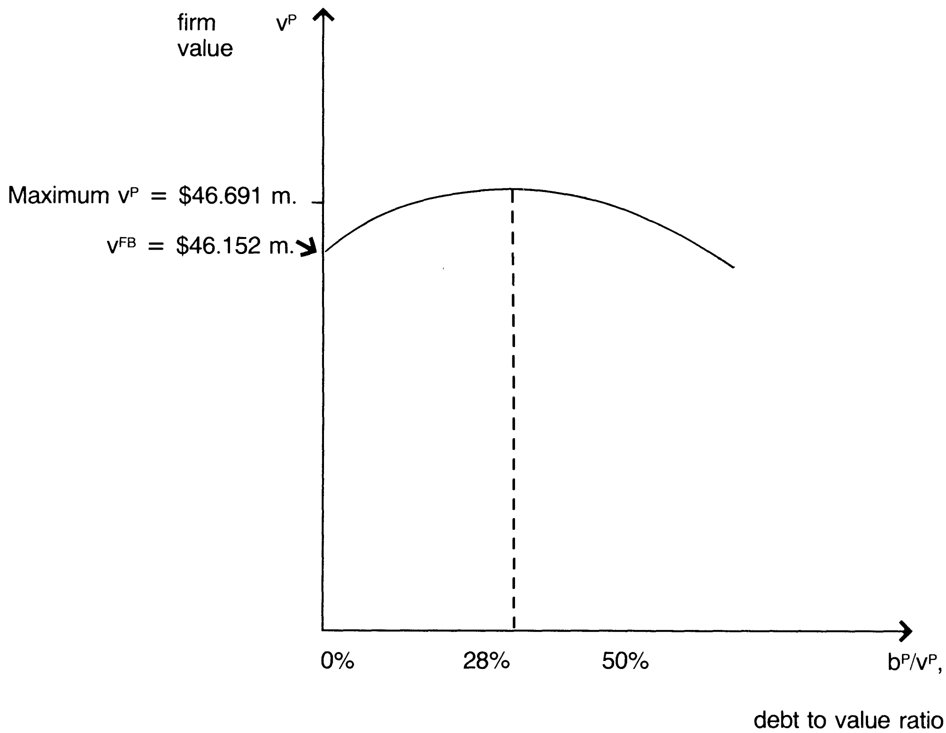

Figure 2. The effect of debt on the value of the firm at a higher current commodity price, s = $1.00 / pound. The value of a levered firm owning a mine with parameters specified in Table I. The bond outstanding has a maturity of 15 years and a coupon rate of 2%. As the constant annual debt service payments are increased the debt-to-value ratio increases. The value of the levered firm initially increases due to the marginal benefits of interest tax shields. As the debt-to-value ratio further increases the marginal agency costs rise and the value of the firm begins to fall.

Consider first the possibility of lengthening the time to maturity on the bond while keeping the present value of the bond constant. In Panel A of Table VII we show this comparison. At an initial commodity price of $0.45/pound and a firm with an outstanding bond with annual debt service payments totalling $0.5 million for 5 years faces a high probability of bankruptcy. The levered firm has a value of $1.602 million—compared to a first best value of $6.9 million. The market value of the bond is $1.374 million, a discount of 42% from par value. It is possible to design a bond with longer maturity, lower total debt service, and with an approximately equivalent market value: 15 years, $0.047 annually, and current market value $1.390. If this longer maturity bond is substituted or exchanged for the shorter maturity bond, then the value of the firm rises by over $5 million or more than three times. While the tax shields on the two bonds are almost equal, the longer maturity bond has significantly lower agency costs, the difference being the difference in firm value of $5 million. The shorter

Table VII

The Effect of Maturity Length on Agency Cost

Panel A: Initial Commodity Price, s = \$0.45/pound

Final Maturity Date	Annual Bond Payments	Levered Firm Value $v^P(s, Q, t; 1)$	Bond Value $b^P(s, Q, t; 1)$	Tax Shields $\eta^P(s, Q, t; 1)$	Agency Costs $\psi^P(s, Q, t; 1)$
5 years	0.5	1.602	1.374	0.30	5.598
15 years	0.047	6.836	1.390	0.31	0.374

Panel B: Initial Commodity Price, s = \$0.65/pound

Final Maturity Date	Annual Bond Payments	Levered Firm $v^P(s, Q, t; 1)$	Bond Value $b^P(s, Q, t; 1)$	Tax Shields $\eta^P(s, Q, t; 1)$	Agency Costs $\psi^P(s, Q, t; 1)$
5 years	1.75	15.209	8.212	0.164	1.095
15 years	0.95	13.581	8.207	0.478	3.037

maturity bond with its attendant high nominal debt service requirements puts the firm in imminent danger of bankruptcy. The equity holders' call option on the firm is way out of the money, and they are unlikely to want to continue to pay the debt service and maintenance costs in order to maintain their option. Only a very quick rise in prices will keep the firm out of costly bankruptcy. The longer maturity bond has lower debt service and an effectively longer time to maturity on the equity holders' call option. There is an attendant greater probability that their option will finish in the money, and consequently a lower probability of costly bankruptcy.

Extending the maturity of the bond and lowering the debt service payments does not always increase the value of the firm. When the firm is far from bankruptcy the owners will pay off the short-term bond and quickly return to a first best operating policy. In the meantime, if the term of the bond is short, then the total variance in price is relatively small, and the firm is likely to continue open, never making a suboptimal closing decision. If the maturity of the bond is lengthened, the firm will operate levered for a longer period of time, and the total variance in the price during this term is correspondingly greater. The firm is more likely to hit the close and open trigger prices repeatedly. While the firm is levered it will be opening and closing according to less than first best policy, lowering the value of the firm. The increased time to maturity also increases the possibility that the price will fall far enough to induce costly bankruptcy. A shorter maturity bond forces the equity holders to make the exercise decision relatively quickly when their option is in the money, and hence avoids this costly bankruptcy choice. This case is demonstrated in Panel B of Table VII. At an initial commodity price of \$0.65/pound, and with an outstanding bond-paying annual debt service of \$0.95 million for 15 years, the firm has a value \$13.581

million. The bond has a market value of $8.207 million. The maturity of the bond could be shortened to 5 years with annual debt service payments totalling $1.75 million, and its value would be approximately the same at $8.212. The value of the firm would rise to $15.209 million because the agency costs would have fallen by nearly $2 million.

IV. Conclusion

In this paper we have shown how to adapt a contingent claims model of the firm to reflect the incentive effects of the capital structure and thereby to measure the agency costs of debt. An important feature of our model is the existence of an underlying analysis of the firm and the stochastic features of its product market. We solve directly for the operating policy that is optimal for the equity owners and compare this with the first best operating policy. Our measure of the agency costs of debt is directly related to this underlying change in the use of the productive assets. The model determines the value of the firm and its associated liabilities incorporating the agency consequences of debt as well as the tax benefits.

Extending the agency literature to incorporate contingent claims techniques enables us to make a more refined use of the insights developed in that literature. The contingent claims technique yields a measure of agency costs that is robust to variations in the underlying parameters including the stochastic variable determining the firm's value. This measure can then be used to compare different capital structures and to analyze the agency effects under different circumstances facing the firm. We have already illustrated in this paper how the model can be used to compare the size of the agency costs associated with alternative maturity lengths of comparable debt instruments, depending upon the price of the firm's product. We also believe that this model can be extended to analyze debt contracts of fundamentally different design.

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